Effect of contact resistance in solid-state thermionic refrigeration

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An analytical model of thermionic emission cooling that includes contact resistance is presented. The electrical current density necessary for peak operation of thermionic emission coolers is such that even the slightest resistance in the contacts to the devices will significantly reduce the cooling and coefficient of the performance. The effect of contact resistance is analyzed numerically using a model of thermionic emission cooling based on Fermi–Dirac statistics. The cooling and coefficient of performance are shown to be reduced dramatically by even the slightest contact resistance. [DOI: 10.1063/1.1481777]

I. INTRODUCTION

Although several models of thermionic emission cooling have been developed,[1–5] none has analytically modeled the effect of contact resistance. This is an important consideration because ohmic contacts to a thermionic emission device will have non-zero electrical resistance.[6] Due to the high electrical current densities necessary for the operation of thermionic emission coolers,[2,3] Joule heating at the contacts will have a significant effect on the cooling ability of a thermionic devices.[7] To the knowledge of the authors, the only consideration of contact resistance in thermionic emission cooling has been by LaBounty *et al.* through numerical simulations[7,8] and experimental results by Fan *et al.* that indicated the importance of contact resistance.[9]

For comparison, consider the effect of contact resistance on thermoelectric devices.[10–13] Cooling in a Bi_2Te_3 device with a thermoelement length of half a millimeter will be reduced less than 5% by a contact resistance of $10^{-6}\ \Omega\ \mathrm{cm}^2$. Larger devices are affected even less. Thus contact resistance has only a minor effect on thermoelectric devices. In contrast, it will be shown in this article that the contact resistance can dramatically reduce the cooling capability of thermionic devices.

II. IDEAL CURRENT CHARACTERISTICS

Figure 1 shows a band diagram of a thermionic emission cooler with an applied bias. T_C refers to the temperature of the emitter–barrier junction, the cold side, and T_H refers to the temperature of the barrier–collector junction, the hot side. Other variables shown in Fig. 1 are explained below.

The electrical current density, J_E, and the heat current density, J_Q, in terms of the Fermi–Dirac integrals, $\mathcal{F}_n(\eta)$, through a thermionic emission device are[4]

$$J_E = A^* T_C^2 \mathcal{F}_1(\eta) - A^* T_H^2 \mathcal{F}_1\left(\eta \frac{T_C}{T_H} - \frac{qV}{kT_H}\right), \tag{2.1}$$

$$\begin{aligned} J_Q = {} & A^* T_C^3 \frac{k}{q}[2\mathcal{F}_2(\eta) - \eta \mathcal{F}_1(\eta)] \\ & + - A^* T_H^3 \frac{k}{q}\left[2\mathcal{F}_2\left(\eta \frac{T_C}{T_H} - \frac{qV}{kT_H}\right) \right. \\ & \left. - \eta \frac{T_C}{T_H}\mathcal{F}_1\left(\eta \frac{T_C}{T_H} - \frac{qV}{kT_H}\right)\right] - \frac{\kappa_l}{d}\Delta T, \end{aligned} \tag{2.2}$$

where A^* is the effective Richardson constant defined as

$$A^* = \frac{4\pi q m^* k^2}{h^3}, \tag{2.3}$$

where k is the Boltzmann constant, m^* is the electron effective mass and h is Planck's constant. η is referred to as the reduced Fermi energy and is exactly defined as

$$\eta = \frac{\varepsilon_f^{\text{emitter}} - \varepsilon_C^{\text{barrier}}}{kT}, \tag{2.4}$$

as shown in Fig. 1 where $\varepsilon_f^{\text{emitter}}$ is the Fermi level in the emitter and $\varepsilon_C^{\text{barrier}}$ is the conduction band edge of the barrier at the emitter–barrier junction. V is the applied voltage, κ_l is the lattice thermal conductivity of the barrier material, d is the width of the barrier layer, and $\Delta T = T_H - T_C$. It is assumed that the chemical potential of the barrier is independent of the temperature.

To determine an upper limit of thermionic performance, the barrier width is set to one mean free path. This is the upper limit for the assumption that transport through the barrier is ballistic, an assumption on which this model is based.

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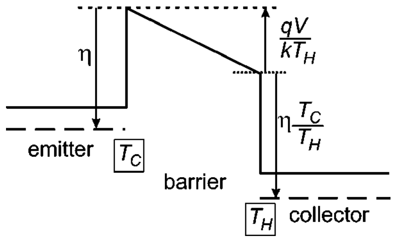

FIG. 1. Band diagram of a thermionic emission cooler with a voltage applied showing the reduced Fermi energy at either side of the barrier region.

III. INCLUSION OF CONTACT RESISTANCE

Figure 2 shows an electrical circuit for a thermionic device with resistive contacts as well as a schematic describing the thermal environment.

The electrical current density is modified by a difference in voltage across both the emitter and collector contacts. The applied voltage, V_{app}, is

$$V_{\mathrm{app}} = V_b + 2 r_c J_E(V_b), \tag{3.1}$$

where V_b is the voltage drop across the barrier, $J_E(V_b)$ is the electrical current density defined by Eq. (2.1) for forward bias and r_c is the contact resistance, taken to be the same for both the emitter and collector contacts.

It is assumed that the thermionic device is ideally packaged, meaning that the emitter (the cold side) is in perfect thermal isolation from the environment and the collector is in perfect thermal contact with the environment. With this idealization, any heat generated at the emitter contact flows to the emitter–barrier junction where the cooling occurs, whereas heat generated at the collector contact freely flows to the environment without affecting device performance. Thus the net heat current density, J_Q^{net}, is affected by only the emitter contact:

$$J_Q^{\mathrm{net}} = J_Q(V_b) - J_E(V_b)^2 r_c, \tag{3.2}$$

where $J_Q(V_b)$ is the heat current density through the ideal device defined by Eq. (2.2).

Maximum cooling is determined by setting the net heat current density to zero and numerically solving for the temperature difference:

$$J_Q^{\mathrm{net}}(\Delta T_{\max}) = 0. \tag{3.3}$$

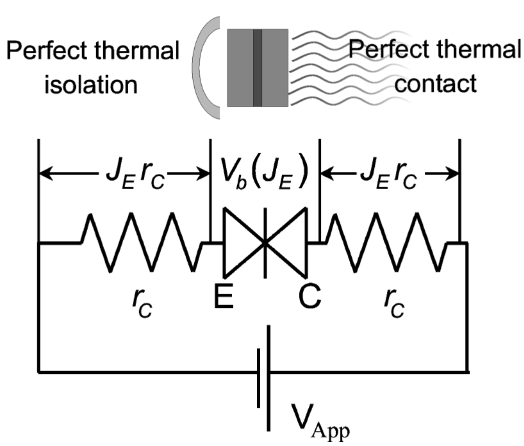

FIG. 2. Thermal environment and electrical circuit for a thermionic emission cooler with contact resistance.

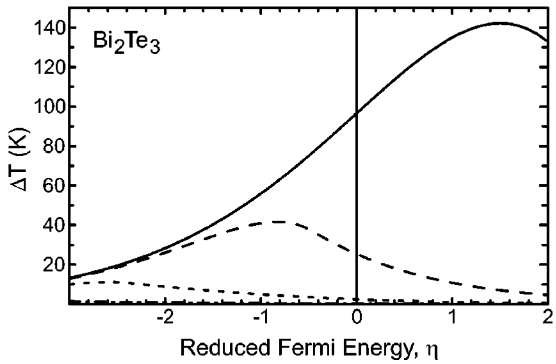

FIG. 3. Maximum cooling as a function of the chemical potential with contact resistances of 0 (solid line), 10^{-8} (dashed line), 10^{-7} (dotted line), and 10^{-6} $\Omega\,\mathrm{cm}^2$ (dash-dotted line) for a Bi_2Te_3 device.

Figure 3 shows the maximum cooling as a function of the reduced Fermi energy for the ideal case and for three contact resistances (10^{-8}, 10^{-7} and 10^{-6} $\Omega\,\mathrm{cm}^2$) in a thermionic device with a Bi_2Te_3 barrier. With a resistance of 10^{-6} $\Omega\,\mathrm{cm}^2$, the cooling capability of a Bi_2Te_3 thermionic emission cooler is reduced from nearly 140 to about 2 K.

The coefficient of performance (COP), defined as the ratio of the heat removed from the cold junction to the electrical power used in the device,

$$\mathrm{COP} = \frac{J_Q^{\mathrm{net}}}{J_E V_{\mathrm{app}}}, \tag{3.4}$$

is likewise effected. Figure 4 shows the maximum COP and the percentage of Carnot COP for the ideal case and for three contact resistances (10^{-8}, 10^{-7} and 10^{-6} $\Omega\,\mathrm{cm}^2$) in a thermionic device with a Bi_2Te_3 barrier. All four curves appear, however, the curve for 10^{-6} $\Omega\,\mathrm{cm}^2$ is barely visible. Such contact resistance reduces the COP to 0.3% of the ideal COP.

IV. CONCLUSION

An analytical model of thermionic emission cooling that includes contact resistance was presented and has revealed that the effect of contact resistance is severe. Whereas even moderate contact resistance (for example, 10^{-6} $\Omega\,\mathrm{cm}^2$) has virtually no effect on a bulk thermoelectric device, it reduces

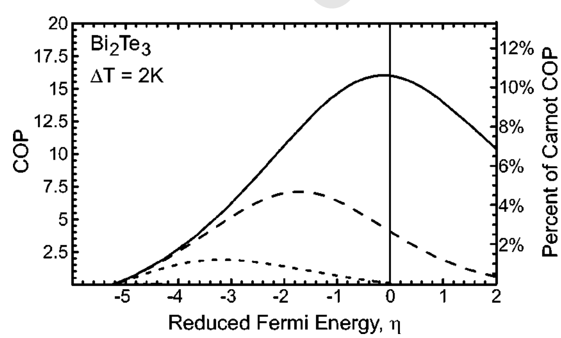

FIG. 4. Maximum COP as a function of the reduced Fermi energy with contact resistances of 0 (solid line), 10^{-8} (dashed line), 10^{-7} (dotted line), and 10^{-6} $\Omega\,\mathrm{cm}^2$ (dash-dotted line) for a Bi_2Te_3 device with a temperature difference of 2 K operating at room temperature.

cooling and the coefficient of performance in a thermionic emission device dramatically. To date, the only experimental estimate of a contact resistance to a thermionic device is $1.5\times10^{-7}\ \Omega\,\mathrm{cm}^2$ for a SiGeC/Si thermionic cooler.[9] Even assuming material properties favorable to thermionic emission cooling, our calculations suggest at least an order of magnitude improvement in contact resistance will be necessary to achieve cooling comparable to that of conventional thermoelectric devices.

[1] G. D. Mahan and L. M. Woods, Phys. Rev. Lett. **80**, 4016 (1998).
[2] A. Shakouri and J. E. Bowers, Appl. Phys. Lett. **71**, 1234 (1997).
[3] A. Shakouri and J. E. Bowers, Proceedings of the 16th International Cconference on Thermoelectrics 1997, p. 636.
[4] M. D. Ulrich, P. A. Barnes, and C. B. Vining, J. Appl. Phys. **90**, 1625 (2001).
[5] G. D. Mahan, in *Recent Trends in Thermoelectric Materials Research III, Semiconductors and Semimetals*, edited by T. M. Tritt (2001), Vol. 71, p. 157.
[6] S. M. Sze, *Physics of Semiconductor Devices*, 2nd ed. (Wiley, New York, 1981), p. 304.
[7] C. LaBounty, A. Shakouri, G. Robinson, P. Abraham, and J. E. Bowers, Proceedings of the 18th International Conference on Thermoelectrics, 1999, p. 23.
[8] C. J. LaBounty *et al.*, in *Thermoelectric Materials 2000—The Next Generation Materials for Small-Scale Refrigeration and Power Generation Applications*, edited by T. M. Tritt, Mater. Res. Soc. Symp. Proc. **626**, p. Z14.4.1
[9] X. Fan *et al.*, Appl. Phys. Lett. **78**, 1580 (2001).
[10] G. Min and D. M. Rowe, Energy Convers. Manage. **41**, 163 (2000).
[11] C. M. Cortes and R. G. Hunsperger, IEEE Trans. Electron Devices **ED-27**, 521 (1980).
[12] G. Min and D. M. Rowe, Solid-State Electron. **43**, 923 (1999).
[13] Y. S. Ju and U. Ghoshal, J. Appl. Phys. **88**, 4135 (2000).

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